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MiNI BLUEBIRD MATH CIRCLE Issue17: Slide Calculators

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https://akademia.mini.pw.edu.pl/pl/ukraina

A logarithmic number line might be an intuitive mathematical concept, whereas the idea of a linear number line might have to be learned.
-Stanislas Dehaene, neuroscientist


MATH COYOTE CORNER
"What is $2 \times 2$ ?"
The engineer whips out a slide rule, shuffles it back and forth, and finally announces, "3.99"

The mathematician ponders for a while, then says: "I don't know what the answer is, but I can tell you, an answer exists!"

## Inspiration: Traci Sorell's Classified

The acclaimed Cherokee author Traci Sorell wrote Classified. The book is a biography of Mary Golda Ross, a Cherokee aerospace engineer, mathematician, and "hidden figure." Mary Golda Ross worked on many classified projects-and much of what she did remains a secret to this day. Yet her work was of tremendous importance to the US space program, and she broke barriers as a Cherokee woman. Traci Sorell also answered questions from Bluebird Math Circle participants. You can see more here:
https://aimathcircles.org/issue-25-traci-sorell-reads-classified/


## Family Circle: Make Your Own Slide Calculators

On the cover of Classified and on illustrations within, Mary Golda Ross is holding her slide rule. The slide rule is an elegant computing tool. Depending on its design, a slide rule can help you work out sums and products, or compute special functions such as logarithms and trig. So can computers. Why do people still love slide rules in our day and age? The power of estimation!

Get a taste of this with a few quick experiments below. We describe them with paper cards, but any paper will work. Mathematicians and scientists often design their own tools, physical or virtual (such as software).

## Activity 0. Paper strips

Fold a sheet of paper in half, and then in half again. Cut along the folds to make paper strips.


## Activity 1. Addition slide rule

Take a paper strip, and draw a line through the middle. Use your thumb or a ruler to mark equal units across the line.

Label the marks $0,1,2,3 \ldots$ on the top AND the bottom of the line. Then cut along the line. You will have your own addition slide rule, made of two slides: top and bottom.

Suppose you want to add $3+5$. Move the bottom slide until the bottom 0 lines up with top 3 . Now find 5 on the bottom slide. It is directly under 8 , which is your answer! Try with your own example. Can you explain why your addition slide rule works?

## Activity 1.5 Adding the numbers in-between

The marks on your slide rule are discrete. That is, you need to jump to get from one mark to another. But the lines themselves are continuous. That is, you can smoothly slide from point to point along the lines without jumping. Continuous lines present a great opportunity to make your slide rule more powerful! Imagine the numbers in-between your marks, such as $1 / 2,64 / 5,3.7$, etc.

## Activity 1.999 What about multiplication?

If you have time to explore, try this on your own before the next guided activity. Now that you have a slide rule that adds, can you build a similar one that multiplies? Would it work in the same way?

## Activity 2. Even more power(s)

This slide rule's trick comes from a clever link between addition, multiplication, and powers. Building this slide rule can help you understand the link. Get a new paper strip. As before, draw a line through the middle of your strip. Use your ruler to mark equal units
 across your line, and label them $0,1,2,3 \ldots$ On this slide rule, the labels stand for exponents in powers of 2 . Next to each label, write out the exponents: $2^{0}, 2^{1}, 2^{2}, 2^{3} \ldots$ (Reminder: $2^{3}$ means $2 \times 2 \times 2$.) Finally, calculate these powers, write them on your card, and circle them: $1,2,4,8 \ldots$
Repeat the previous example, $3+5$, using this power slide rule. Move the bottom slide until the bottom label 0 lines up with top label 3 . Find 5 on the bottom slide. As before, it lines up with 8 at the top, because $3+5=8$. Now look at the circled powers next to 3,5 , and 8 . They are 8,32 , and 256 . In fact, $8 \times 32=256$. This slide rule multiplies the circled powers! Try to multiply powers with your own examples. Can you explain why the circled powers multiply when you add their exponents?

## Activity 2.5 Multiplying numbers in-between

What if you want to multiply numbers that aren't $1,2,4,8 \ldots$ and other powers of 2 ? Continuity to the rescue! Where do you think $1 / 2,3,5,6,7.2$ and other numbers in-between powers of 2 should be on your powers slide rule? Why? Is it possible to use your powers slide rule to estimate products of fractions and decimals? How?

## High school challenge: A function of your choice

If you are into algebra, you can make a slide rule for many functions, such as $f(x)=x^{2}$ or $f(x)=\log x$. How? This is a big exploration in itself! Use a graphing calculator such as https://www.desmos.com/calculator to help you find the values of your function and "the numbers in-between."

Try a virtual slide rule similar to the professional tool Mary Golda Ross used: https://www.sliderules.org/react/raven.html

## Ask Bluebird

QUESTION-A student said today he wants to be an elementary teacher because he wants to teach fractions. He asked: "Why can't people 'get' fractions?" From Donna Fernandez

BLUEBIRD SAYS—What a noble desire! Kudos to your student! Fractions are hard. I hope he offers his future students a lot of hands-on, modeling activities about fractions. Too many elementary school activities feature discrete models. In discrete models, we count separate whole objects: 2 pencils, 10 coins, 5 chairs, and so on. Fractions call for a different mindset! We need continuous models that aim for the numbers in-between. Fractions are hard for many people because students don't have enough experience measuring, approximating, and computing non-whole quantities. Fewer than one in twenty US adults can come up with a real-life situation
 where you divide $1 / 2$ by $4 / 3$. For example: what is your average miles/hour speed if you walk half a mile in an hour and twenty minutes?

FUN FACT OF THE FORTNIGHT Go outside on a sunny day and observe shadows. They are shorter when the sun is high around noon. They are longer when the sun is low over the horizon. Stretching and shrinking shadows make a great continuous model for fractions! That's why many traditional cultures such as the Navajo or Pueblo people used shadows to measure time, as fractions of days or years. You can build a "sun rule" that multiplies and divides fractions. It works like a slide rule, but with the sun and shadows.

Photo: a modern sun rule by Justin Dimmel and Eric Pandiscio


