Faculty of Mathematics and Information Science
WARSAW UNIVERSITY OF TECHNOLOGY

MiNI BLUEBIRD MATH CIRCLE

Issue 11: Around Pi

Share your problems, solutions, models, stories, and art:
https://akademia.mini.pw.edu.pl/pl/ukraina

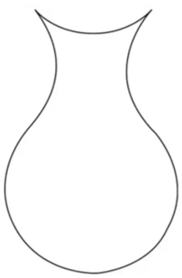
I do not think the measure of a civilization is how tall its buildings of concrete are, but rather how well its people have learned to relate to their environment and fellow man.

— Sun Bear, Chippewa

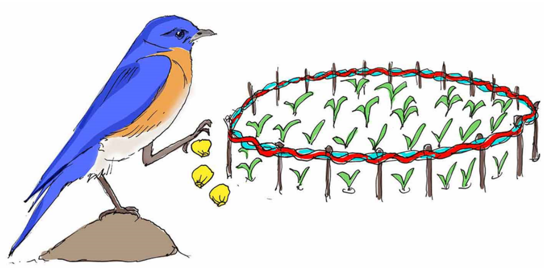
NEWSFLASH Join LIVE MiNI Bluebird Math Circle to work on these activities together with friends and family. Monday March 6, 18:30-20:00 Warsaw, Poland time, online. Sign up at https://akademia.mini.pw.edu.pl/pl/ukraina

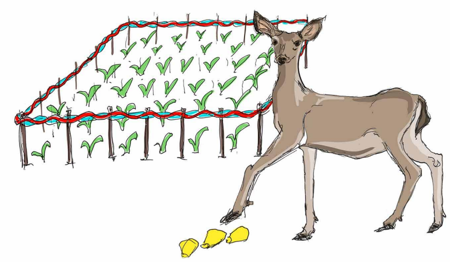
MATH COYOTE CORNER
Q: What do you get if you divide the circumference of a jack-o-lantern by its diameter?
A: Pumpkin pi.

Squaring the Circle



Warm-Up Puzzle With two straight line cuts, divide the vase into three pieces that can be reassembled to form a square.





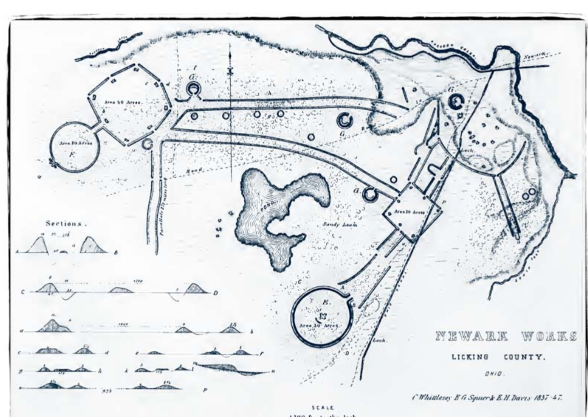
Two Bluebird and Deer Stories **Story 1.** One day Bluebird went to his favorite MathMagic Store and found there very special seeds of corn-with-square-roots. He bought lots of seeds and decided to plant them in a new field. Since Bluebird likes circles, he designed his field as a perfect circle, and put around a colorful rope as the field boundary. Then he thought of Deer, his best friend, and he gave her the remaining seeds (and there were a lot of them) and exactly the same amount of rope he used to enclose his field. Unlike Bluebird, Deer prefers squares. So she made a square field surrounded by all the rope given to her by Bluebird.

Q1: How did she construct a field of the desired shape and size? **Q2:** Both friends planted and harvested their fields for many years, and every year Bluebird's harvest was bigger. Why was it happening?

Story 2. One day Bluebird baked a pie for Deer. Of course, it was a perfectly circular pie. Deer liked it a lot and decided to return the favor. She wanted to make a square pie of exactly the same size.

Q3: How could she make it, provided that she wanted her pie to have the same thickness as Bluebird's one?

Hopewell Culture National Historical Park

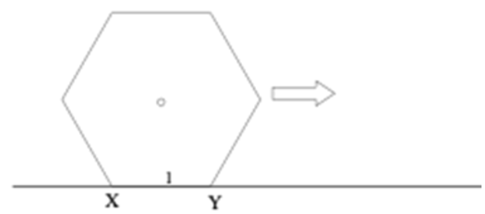


1837 survey of the Newark Earthworks http://touringohio.com/history/great-circle-square.html

Real Life Story The Hopewell tradition describes a network of Native American cultures that flourished in settlements along rivers in the Eastern Woodlands from 100 BCE to 500 CE. Today, the best-surviving features of the Hopewell tradition era are the earthwork mounds. The mounds typically have various geometric shapes and rise to impressive heights. The Newark Earthworks in Ohio are the largest set of geometric earthen enclosures in the world. Scientists believe that they were part cathedral, part cemetery, and part astronomical observatory. Professors Ray Hively and Robert Horn were the first researchers to analyze numerous lunar sightlines at the Newark Earthworks and the High Banks Works in Chillicothe, Ohio. They also found some remarkable geometric facts. In particular, at the former site they described at least three cases of 'squaring the circle', when the area of a circle is equal to the area of a square. There were similar instances at the latter site, too. And the circles (and squares) are huge – the largest of these circles' diameter is 1316 ft. In their book *Encountering Hopewell in the Twenty-first Century, Ohio and Beyond* they write: "Work with the basic shapes encountered in spinning, weaving, basket making, and construction demanded solutions such as how to achieve the space available in a circular dwelling, in a square one, or how to craft a coiled, circular cover for a basket with a rectilinear plaited base. The evidence that the Hopewell more than met these demands is on the ground at Newark." The book: https://ideaexchange.uakron.edu/encountering_hopewell/

A Video Watch the Numberphile video "Squaring the Circle" https://youtu.be/CMP9a2J4Bqw.

Family Circle: Rolling Hexagon



A regular hexagonal wheel is resting on its side labeled XY, as shown in the diagram below. All sides have a length of 1. Suppose the hexagon rolls forward (kathump kathump) in a straight line until the next time side XY is back on the line.

A) What is the length of the path traveled by the center/hub of the wheel?

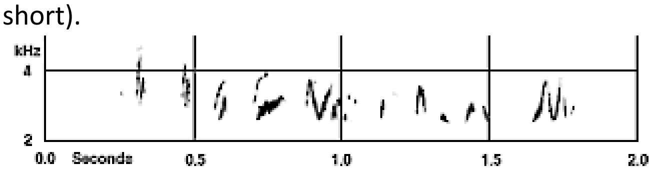
B) What is the length of the path traveled by vertex Y?

C) What shape should the road have so that the wheel rolls smoothly?

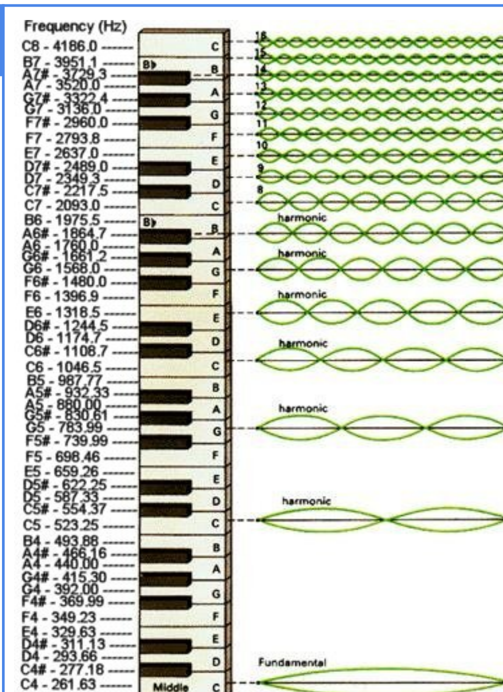
Ask Bluebird

QUESTION—*What is the range of notes in your song? What scale do you use, a five or eight-step scale?* From Chadd McGlone

BLUEBIRD SAYS—The graph below shows one of my (a western bluebird's) songs, with notes between about 2000 and 5000 Hertz (2-5 kHz for short).

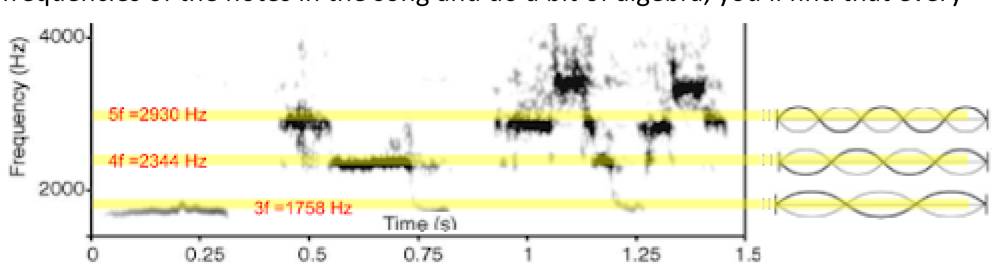


Sound is the vibration of the air. The faster the vibration, the higher the sound. For a sense of that Hertz scale, compare my song to music. For example, the middle C note on the piano is about 260 Hertz, but the highest, rightmost C on the grand piano vibrates about 16 times faster than that, at about 4 kHz. My song, violins, or flutes can also go this high. Out of the human musical instruments, only cymbals or electronic synthesizers can ding and beep as high as the top of my song at 5000 Hertz! I could not find any literature connecting my song to any musical scales, but please read about my friend the hermit thrush in the Fun Fact of the Fortnight.



Song: *birdsoftheworld.org/bow/species/wesblu/cur/sounds* *Graph:* *universe-review.ca/R12-03-wave02.htm*

FUN FACT OF THE FORTNIGHT The **hermit thrush** sings many songs in **harmonic** (or **overtone**) **series** - like a bugle or other simple brass instrument. If you measure the frequencies of the notes in the song and do a bit of algebra, you'll find that every single one of them is a multiple of the same base frequency! For example, if the base frequency **f**=586 Hz, the thrush may sing at **3f**=1758 Hz, **4f**=2344 Hz, and **5f**=2930 Hz. Harmonic series is a big part of human musical scales (including both the five-step and eight-step scales).



Graph of thrush song frequencies adapted from *pnas.org/doi/full/10.1073/pnas.1406023111*