

MINI BLUEBIRD **MATH CIRCLE** Issue7: **The Art of Factorization Diagrams**

All powers have two sides, the power to create and the power to destroy. We must recognize them both, but invest our gifts on the side of creation.

Faculty of Mathematics and Information Science

Share your problems, solutions, models, stories, and art: https://akademia.mini.pw.edu.pl/pl/ukraina

-Robin Wall Kimmerer Braiding Sweetgrass: Indigenous Wisdom, Scientific Knowledge, and the Teachings of Plants

NEWSFLASH Join LIVE the MiNI Bluebird Math Circle to work on these activities together with friends and family. The math circle is in English and Ukrainian with live translation.

Monday November 14th, 18:30-20:00 Warsaw, Poland time, online. Sign up at https://akademia.mini.pw.edu.pl/pl/ukraina

MATH POEM Word Crunching by Brian Bilston

T wrote a poem on a page but then each line grew

by the word sum of the previous two

until I started to worry at all these words coming with such frequency

because, as you can see, it can be easy to run out of space when a poem gets all Fibonacci sequency

Family Circle: The Art of Factorization Diagrams

PROJECT—Try your hand at a maker activity popular with crafters, artists, infographic designers, and cartoonists. The goal is to build your own version of **factorization diagrams**: the number portraits that expose hidden patterns through visual storytelling.



PROMPT: The portrait of a number Take a look at these portraits of number two. Then sketch a portrait of two in your own way.

Did you choose to show two-ness in some way, like two dots? Or did you pick a symbol for two like Sondra's blue square? Interesting choice either way! Figurative art shows physical objects. Abstract art or abstract math symbols stand for ideas. The digit 2 is abstract: it does not resemble two eyes or two leaves.

Now that you portrayed two, also draw three in your style.

"Two" by Richard Schwartz, Todd Lehman, Sondra Eklund, Brent Yorgey, and Nicholas Rougeux. "Color Bar Sweater" by Sondra Eklund.

PROMPT: A composite portrait Let's do some hands-on modeling. Take 6 coins and share them among some people. For example, you could share them among 2 people by giving each 3 coins, or among 3 people by giving each 2 coins. In our story, you wouldn't share the coins among 6 people, because only 1 coin per person feels a bit sad. In fact, here are our sharing rules:

RULE 1: Be fair and always make equal shares.

RULE 2: Be generous and always make each share greater than one.

These rules model our factorization. We call 2 and 3 factors of 6 and say, "Two multiplied by three is six." Now use the portraits of two and three you made before to **compose** a portrait of six.

- Interesting Choice 1: Do you portray six itself, the **product** of your multiplication—or do you 0 only portray two and three and leave multiplying as a puzzle for your art fans?
- Interesting Choice 2: If you do show multiplication and the product, how? Designers often Ο model multiplication via groups, grids, symmetric arrangements, or trees.

PROMPT: Factors of factors Go ahead and make portraits of numbers 4, 5, 6... To portray each number, start by factoring it using Rules 1 and 2. Then use your portraits of these smaller factors to make the composite portrait of your composite number, just like you used the portraits of 2 and 3 to compose the portrait of 6. Wait a minute! Will that always work?



- You will find that you can't make both Rules 1 and 2 work with some numbers. If you have seven things, you either have to keep all seven together, or share seven ways, with each share being a single thing. Such numbers are called **prime numbers**. How will you portray your primes in your factorization diagrams?
- You will also find that some numbers factor many, many ways. 12 is 2×6, 3×4, 2×2×3, and all the ways you can reorder these factors like 2×3×2. Mathematicians talk about **highly composite numbers** or dramatically call them **antiprime**.
- Some composite numbers are products of the same factors, like 16=2×2×2×2. These numbers are called **powers** of their prime factor: sixteen is the fourth power of two.

Once you sketch the first dozen or so numbers, pause and admire your factorization diagram. Exchange your sketches with other diagram makers. Discuss what you like about them, explore their mathematical features, and make more diagrams. Is your design abstract or figurative? Can you see the products and multiplication, or only factors? Are prime numbers, composite numbers, and powers easy to see, or hidden? Visit the factorization diagram gallery at *NaturalMath.com/factorization/*



Ask Bluebird

QUESTION—What is the Bluebird's favorite mathematical equation? From Mary Clarke

BLUEBIRD SAYS—What a sweet question! Thank you for your care. A psychologist recently told me that when we pick a favorite equation, curve, or another math object, the act generates a bit of love. A quirk of our minds (a cognitive bias) makes that object special. If we get in the habit of asking one another for favorites, the math love adds up over time. That's one way to put cognitive biases to good use. Here are several current favorites:



- $r = 1 cos\theta$ because it draws a little heart (a cardioid) in polar coordinates
- $(x^5 + x^4 + x^3 + x^2 + x + 1)(x 1) = x^6 1$ because you multiply a big long thing by x 1 and get something very nice and easy
- $\circ e^{\pi i} + 1 = 0$ because it connects the five most important (or famous) numbers in just one equation
- \circ 1/2 + 1/3 + 1/6 = 1 because it's magic that three different unit fractions can add up to 1
- $\circ \quad \infty \, + \, 2 \, = \, \infty$ because it's so different from what the numbers do



FUN FACT OF THE FORTNIGHT Take some square graph paper, number its cells along a spiral, and then color those cells numbered with primes. Zoom out, and colored cells turn into dots. The resulting picture doesn't look random. The dots cluster along diagonal lines. That design is called *Ulam's Spiral*. Similar patterns of lines emerge if we use triangular, hexagonal, or round grids. Nobody knows why these lines appear as they do!

