Faculty of Mathematics and Information Science
WARSAW UNIVERSITY OF TECHNOLOGY

MiNI BLUEBIRD MATH CIRCLE
Issue 3: Chessboard and Tiling

Share your problems, solutions, models, stories, and art:
https://akademia.mini.pw.edu.pl/pl/ukraina

Freedom of free thought—not thoughts of what we have been taught to think. It means throwing away all your old ideas that you've been comfortable with. Fear of the unknown has always been a stumbling block. You must be willing to totally let go of yourself, without fear, and without fear of what you will find.

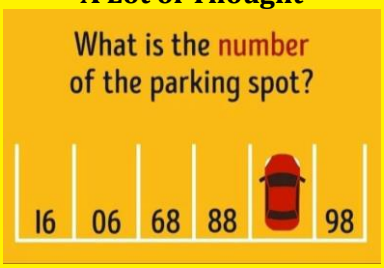
–J.C. Elliott-High Eagle (Osage-Cherokee), physicist, musician, NASA Engineer (retired)

NEWSFLASH Join LIVE the MiNI Bluebird Math Circle to work on these activities together with friends and family. The math circle is in English and Ukrainian with live translation.
Monday August 22nd, 18:30-20:00 Warsaw, Poland time, online.

Sign up at https://akademia.mini.pw.edu.pl/pl/ukraina

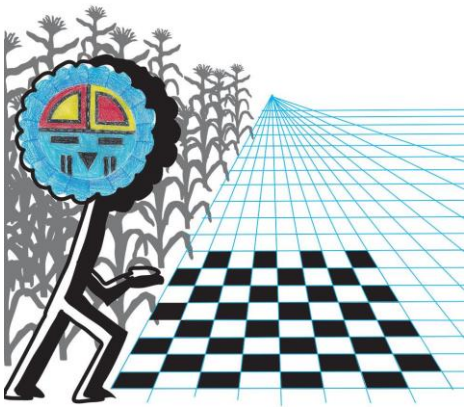
MATH JOKE

A Lot of Thought

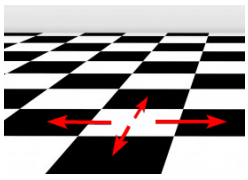


Family Math Circle: Chessboard and Tiling Mathematics

TAWA STORY Planting season was underway, and LaVerne and Jordyn Lomakema (Hopi Junior/Senior High School) thought it fitting to tell the following story.



A rectangular cornfield is painted black and white in a checkerboard pattern, so that it looks like a 100-by-85 chessboard. Tawa (Sun) is standing with a large sack of corn kernels in one corner of the field, and at this point, the field is empty.

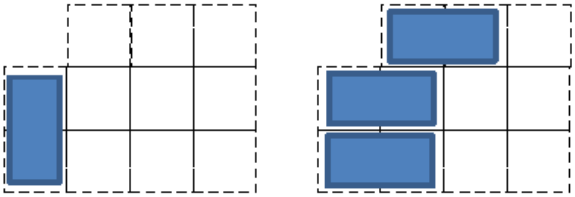


He can take a step from any one of the squares to one of the adjacent squares (up, down, left, or right, but not diagonally). When he moves to a new empty square, he puts a kernel down on it. But if the new square already has a kernel, Tawa removes the kernel.

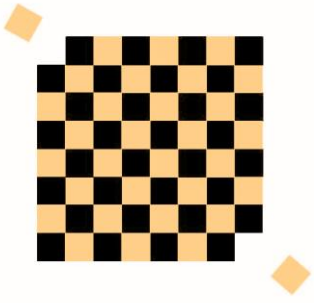
Is it possible for Tawa to walk around the field in such a way that in the end there is exactly one kernel lying in each of the black squares and no kernels lying in the white squares?

Tawa illlustrations by Jordyn L.

MISSING SQUARES Your old 8-by-8 chessboard is missing two squares at opposite corners. You have 31 dominos, and you want to tile what's left of the chessboard with these dominos. A domino is a 2-by-1 tile and so it covers exactly two squares of the chessboard, for example:

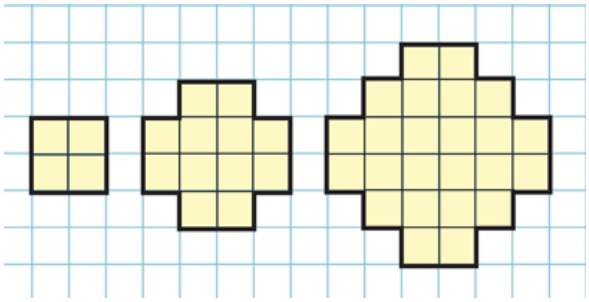


Could you do this?



SPEAKING OF DOMINO TILING Here is an interesting game: count the number of possible ways to tile the following shapes with 2-by-1 dominos. How many ways to tile shape #1? #2? #3? Can you guess what the next shape would be like, and how many ways are there to tile it?

These shapes can be found in much indigenous art. The motif is shared between many nations. Do you recognize it?



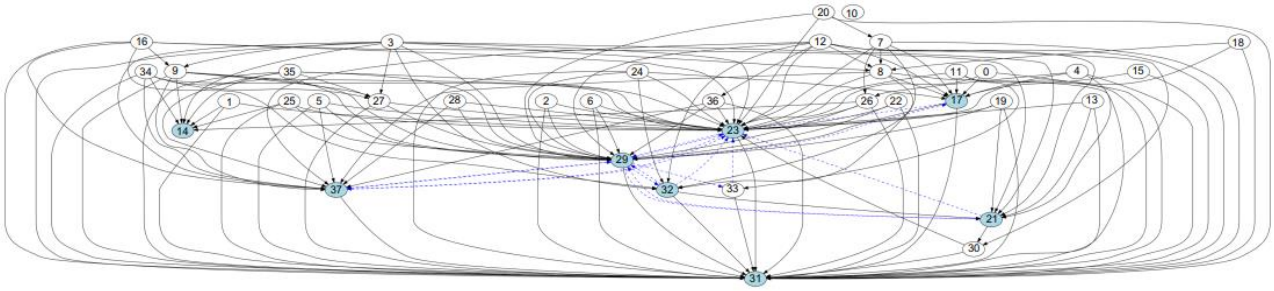
Ask Bluebird

QUESTION—*What does math modeling mean?* From Beth Cammarata, Santa Fe Indian School

BLUEBIRD SAYS—Mathematical modeling is in a sense the opposite of storytelling. We start with a story or real life situation, and strip it of all non-essential details. This reduces - or elevates it - to a level of abstraction which allows us to solve the problem using appropriate mathematical tools.

One example is helping hundreds of thousands of patients in dire need of kidney transplants. In the US, at every given time about 100,000 people are waiting to get a kidney that would save their lives. Quite often each of these people have relatives or friends who are willing to donate their kidney, but they do not match their loved ones.

So there is a list of people in need of transplant, and a list of available kidneys. These lists contain hundreds of thousands of entries. What we need is to find good ‘chains.’ For example, Alan needs a transplant and Jan donates her kidney, but it isn’t compatible with Alan. Could we find appropriate donors and recipients so that Jan’s kidney goes into Pat, Pat’s donor’s kidney goes into Dana, and so on, until some donor’s kidney is transplanted into Alan? In 2007 a team of mathematicians invented a way to produce ‘kidney transplant chains.’ They used graph theory, and their solution has already saved more than 80,000 lives. Here's what a transplant compatibility graph looks like:



Mathematical model by Ross Anderson and colleagues

FUN FACT OF THE FORTNIGHT

According to legend, chess was invented by Grand Vizier Sissa Ben Dahir, and given as a gift to King of India. The king was so delighted that he offered him any reward he requested, provided that it sounded reasonable. The Grand Vizier requested the following: "Just one grain of wheat on the first square of a chessboard. Then put two on the second square, four on the next, then eight, and continue, doubling the number of grains on each successive square, until every square on the chessboard is reached."

Intuitively, King — just like almost anybody else — underestimated the number of grains and laughed at Sissa because he had asked for such a small gift. When he had someone to calculate the total number of grains, it took more than a week before he came back with the solution. King undoubtedly became very pale when he got the answer: the aggregated number of grains on all squares of a chessboard would be 18,446,744,073,709,551,615 grains, or about 2,635,249,155,000 tons of wheat. This is the harvest of all the wheat of the world, for several decades.