# MiNI BLUEBIRD MATH CIRCLE
## Issue 13

# How are Navajo Clans, the Game of SET, and a Faraway Planet the Same?

Share your problems, solutions, models, stories, and art:
[https://akademia.mini.pw.edu.pl/pl/ukraina](https://akademia.mini.pw.edu.pl/pl/ukraina)

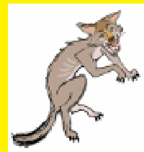
*It's not being afraid to fail. … Maybe you'll stumble and maybe you'll have setbacks along the way, but not letting that fear hold you back. To me, that's true courage.*

—Col. Nicole Aunapu Mann (Wailacki of the Round Valley Indian Tribes), NASA SpaceX Crew-5 mission commander

**NEWSFLASH**

Join LIVE MiNI Bluebird Math Circle to work on these activities together with friends and family. Monday April 24, 18:30-20:00 Warsaw, Poland time, online.

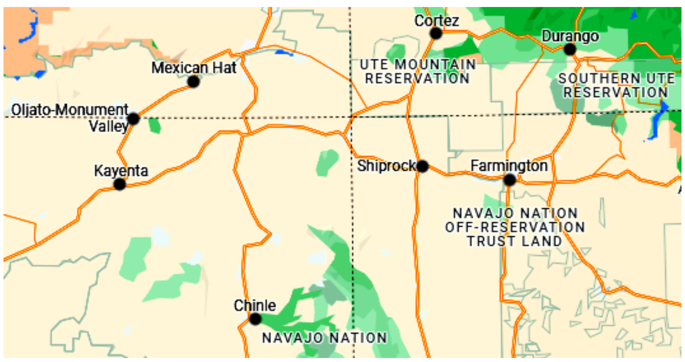
Sign up at [https://akademia.mini.pw.edu.pl/pl/ukraina](https://akademia.mini.pw.edu.pl/pl/ukraina)



**MATH COYOTE CORNER**

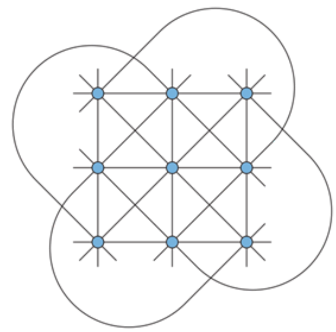
Parallel lines have so much in common. It's a shame they're never gonna meet.

## Inspiration: Google Maps - Towns and Roads




What do we see on the map? If we ignore all the words and color patches what remains will be roads (orange lines) and towns (little circles or points). The picture on the right also consists of lines and points. We can think of this picture as a map of some 'ideal' (= mathematical) place.

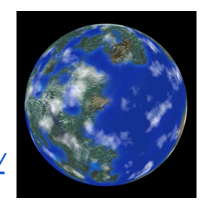
*Image:* [*wikipedia.org/wiki/Finite_geometry*](https://wikipedia.org/wiki/Finite_geometry)



## Family Circle: Points and Lines

Imagine that you're on a space expedition and you discover a faraway beautiful totally uninhabited planet with earth-like physical conditions. There is a big island, and you want to build a number of villages there – you expect that soon many earthlings will be interested in coming to this planet and you want them to be comfortable.

*Image:* [*https://sos.noaa.gov/catalog/datasets/exoplanet-earth-like/*](https://sos.noaa.gov/catalog/datasets/exoplanet-earth-like/)



**Problem I.** You want to start with 4 villages, and you want to connect them with roads. There are some rules you need to observe:

- There is exactly one road between any two villages.
- There are exactly two villages on each road.

Draw a possible map. Use dots to denote villages. Use lines, not necessarily straight, to denote roads.

Questions:

1. How many roads do you have?
2. Name some roads that do not have any villages in common (we will call such roads non-intersecting).
3. How many groups of non-intersecting roads are there?

For **every group** of non-intersecting roads do the following:

- Make all roads in the group intersect at one point. Note: you might need to extend some roads.
- Build a new village at this intersection.
- Connect all new villages with one new road.

Next questions:

4. How many new villages are there?
5. What is the total number of villages?
6. What is the total number of roads?
7. How many villages are there on each road?

**Problem II.** The total deck of cards for the game of SET consists of 81 cards. Each card has 4 attributes – number, shading, color, and shape. Each attribute has 3 values: number can be 1, 2, or 3; shading can be empty, striped, or solid; color can be green, red, or purple; shape can be diamond, oval, or squiggle. We say that 3 cards form a set if for every attribute all three cards have the same value, or all three cards are of different values.

For our purposes today we will consider only 9 cards on the right. Try to find as many sets as you can among these 9 cards. How many did you find? Now, think of each card as a point and draw a line (not necessarily straight) through each set that you've found. Does your picture look familiar?

*SET cards image from* [https://www.quantamagazine.org/set-proof-stuns-mathematicians-20160531](https://www.quantamagazine.org/set-proof-stuns-mathematicians-20160531)

## Finite Geometries

In geometry that you study in school the main characters are points and lines. And there are infinitely many of them. In fact, every line contains infinitely many points.

In Problem I, we want to map some "villages" (points) and "roads" (lines). Then we add points and lines, making bigger maps. These maps model *Finite Planes*, so called because they only have a few points and lines, unlike our usual, infinite plane. If every pair of lines intersect, we have a Finite Projective Plane, or FPP. If there exist non-intersecting (parallel) lines, we have a Finite Affine Plane, or FAP.

The goal of Problem II is to construct another finite plane. Could you see whether it is a FAP or a FPP?

Fun Fact of the Fortnight section contains something amazing about finite projective planes and the power of our computers - read it!

## Another Finite Geometry

When a Navajo person introduces themselves, they name their mother's clan (1st), their father's clan (2nd), their maternal grandfather's clan (3rd), and their paternal grandfather's clan (4th). Thus every Navajo Nation member has 4 'attributes' – like cards in the SET game. But unlike the game where each attribute has only 3 values, the situation here is much more complicated. Originally, there were four Navajo clans. But nowadays, there are more than 100 clans, divided into more than 20 groups. This should lead to a fascinatingly rich geometry. Would you like to invent and study this geometry? Share your thoughts with Bluebird! *Image:* [https://navajowotd.com/](https://navajowotd.com/)


## Ask Bluebird

**QUESTION**— What is bluebird's favorite shape? – from Wesley Hamilton

**BLUEBIRD SAYS**— It's a point! Actually, a point in mathematics isn't a shape since it has neither length nor width, but every shape can be made out of points.

**FUN FACT OF THE FORTNIGHT** Every finite projective plane consists of $\mathbf{n^2+n+1}$ points and the same number of lines; every line contains **n+1** points, and every point lies on **n+1** lines. Such a FPP is said to be of order **n**. It is known that a FPP of any prime and prime power (e.g., 2, 3, 4, 5, 7, 8, 9) order exists. It is also known that for some values of n FPPs of order n do not exist - for example, there are no FPP of order 6 or of order 10. But nobody knows whether or not there is a FPP of order 12. If it were to exist, it would have only contained 157 points arranged into 157 lines. All one needs to do is to find such an arrangement which satisfies a few conditions. And yet the computations needed are so enormous that all the power of modern computers isn't sufficient! *Image: Finite projective plane of order 3, from* [https://puzzlewocky.com/math-fun/from-orthogonal-latin-squares-to-finite-projective-planes/](https://puzzlewocky.com/math-fun/from-orthogonal-latin-squares-to-finite-projective-planes/)