Faculty of Mathematics and Information Science
WARSAW UNIVERSITY OF TECHNOLOGY

MiNI BLUEBIRD MATH CIRCLE

Issue 14: Making Things Equal

Share your problems, solutions, models, stories, and art:
https://akademia.mini.pw.edu.pl/pl/ukraina

From my experience, no one expects you to be perfect on the first try and there is honor in knowing where you're lacking. Take the time to better yourself - to not give up.

—Alexis Keeling, Cherokee Nation, Industrial and Systems Engineer

NEWSFLASH

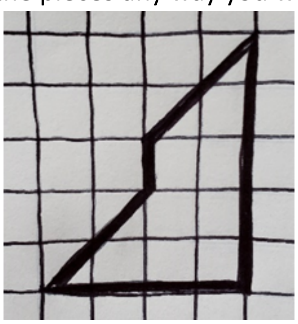
Join LIVE Bluebird Math Circle to work on these activities together with friends and family.
Monday, May 29, 18:30-20:00 Warsaw, Poland time, online.
Sign up
https://akademia.mini.pw.edu.pl/pl/ukraina

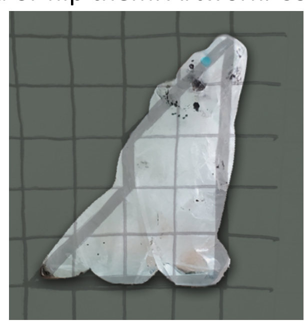
MATH PUZZLE

How many numbers do you see here?

Warm up: Cut It Up Equally

Split the figure on the grid into two equal parts (so that you can place one part on top of the other one and they completely coincide). You can move the pieces any way you want – slide, turn them around or flip them. *Artwork: Coyote (Zuni fetish).*

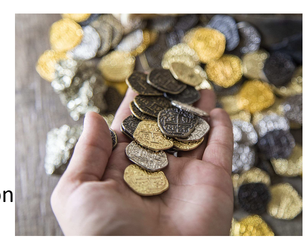




Family Circle: Making Shares or Numbers Equal

Problem 1: Once upon a time on a faraway planet there was an island nation called Bluebird Nest. When the people of Bluebird Nest wanted to appoint the leader, they asked candidates to demonstrate their cleverness, generosity, and fairness. Each candidate was given 100 coins, each coin of different value: 1 blue dollar, 2 blue dollars, 3 blue dollars, etc, all the way to 100 blue dollars, and they were told to distribute the money among people. Whoever distributes the money among the largest number of people in such a way that every person gets the same total value becomes the leader. Could you help?

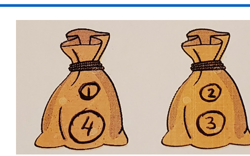
Artwork: *The Money Changer and His Wife* by Quentin Matsys (oil on panel painting, 1514).



Here is an example: If we have three coins of 1, 2, and 3 blue dollars, we can split them evenly between two sacks as shown.

Can we split the coins between more than 2 sacks?

If we have coins of 1, 2, 3, 4 blue dollars, we can split them evenly between two sacks.

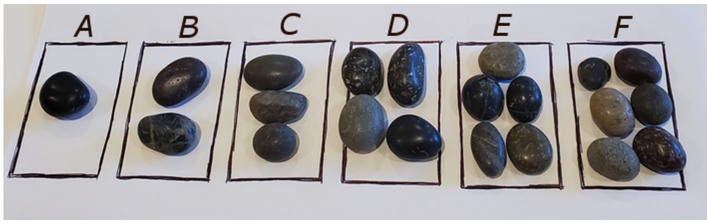


Can we split the coins between more than 2 sacks?

What if there were 101 coins (of 1 blue dollar, 2 blue dollars , 3 blue dollars, …, 100 blue dollars, 101 blue dollars)? What if there were 2023 coins?

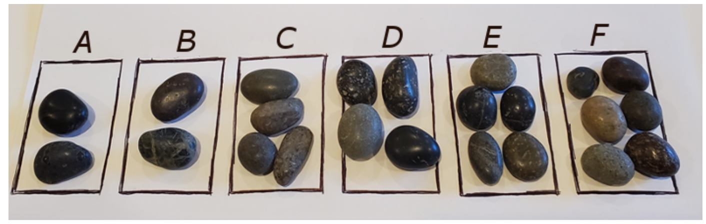
Problem 2: Make the piles equal. The game starts with N piles of stones. The first pile has just 1 stone, the second pile has 2 stones, the third pile has 3 stones, etc. A move consists of adding 1 stone to each of any two piles of our choice (see the example). If we are allowed to make any number of moves, can we end up with all piles having the same number of stones?

EXAMPLE for a move: We start with the piles of size **1, 2, 3, 4, 5, 6, …**



Move: Add 1 stone to the piles A and C.

Now the piles are of the sizes **2, 2, 4, 4, 5, 6, …**



Try to answer the same question if (a) N = 11; (b) N = 12; (c) N = 13; (d) N = 14. Did you notice any patterns?

Ask Bluebird

QUESTION—*What is the smallest perfect number?* – from Chris K.
BLUEBIRD SAYS—A perfect number is one that is equal to the sum of all its divisors including 1 but excluding itself. Let's look at the following table:

Number	All its divisors	The sum of all the divisors excluding the number itself
2	1, 2	1
3	1, 3	1
4	1, 2, 4	3 (=1+2)
5	1, 5	1
6	1, 2, 3, 6	6 (=1+2+3)

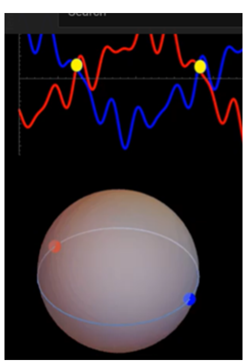
(Can you see why the table starts with number 2 instead of 1?)

From the table we see that the smallest perfect number is 6. Try to find the next perfect number, it won't take long. Many interesting facts are known about perfect numbers. Bluebird's favorite ones are the following two:

1. Nobody knows whether or not there exist odd perfect numbers; this may be the oldest open problem in mathematics.
2. It was the study of perfect numbers that led Pierre de Fermat (a 17th century French mathematician) to discovery of the result (so-called Fermat's Little Theorem) which has recently become totally indispensable in a very applied area of mathematics – cryptography – used everyday in all our electronic devices (telephones, computers, etc.)

FUN FACTS OF THE FORTNIGHT

1. At any given moment on the earth's surface, there exist two antipodal points (on exactly opposite sides of the earth) with equal temperatures and barometric pressures.



2. In geometry, a *polyhedron* (plural *polyhedra*) is a three-dimensional shape with flat polygonal faces, straight edges and sharp corners (or vertices). We are well familiar with many polyhedra such as cubes, pyramids, rectangular boxes. Polyhedra occur in nature as crystals; jewelers shape up stones as intricate polyhedra. We can imagine (or construct) polyhedra of many, many different forms (try it!). But it is a fact that every polyhedron must have at least two faces with the same number of vertices. If you want to see why this is so, write to Bluebird and we'll talk about this fact.